

# Cryptanalysis of The Quantum Secure Direct Communication and Authentication Protocol With Single Photons

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Received: date / Accepted: date

**Abstract** We analyze the security of a quantum secure direct communication protocol equipped with authentication. We first propose a specific attack on the protocol by which, an adversary can break the secret already shared between Alice and Bob, when he (adversary) runs the protocol few times. The attack shows that there is a gap in authentication procedure of the protocol, and by doing so the adversary can obtain the key without remaining any trace of himself. We then give the modification of the protocol and analyze the security of it, and show how the modified protocol can close the gap.

**Keywords** quantum secure direct communication · attack · authentication · single photon · cryptanalysis · security

## 1 Introduction

Over the last decades, quantum cryptography plays a significant role in abstract theory of information and communication security. It is divided into some major research topics, such as QKD <sup>1</sup>, QSS <sup>2</sup>, QIA <sup>3</sup>, etc. which have developed from their first publications [1-3] respectively, until now.

A new topic of quantum cryptography which has been studied at depth, comprehensively recently, is quantum secure direct communication, known as

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<sup>1</sup> quantum key distribution

<sup>2</sup> quantum secret sharing

<sup>3</sup> quantum identity authentication

QSDC in the literature. The goal of QSDC is to convey a secret message directly without a key generating session to encrypt the message. Like many other topics of quantum cryptography, there are two approaches to research on QSDC: quantum entanglement [4], [5], [6], [7], and single photons [8], [9], [10], [11], [12], [13].

Recently many papers have been published in QSDC and some related topics such as quantum dialogue (QD) and quantum secure direct dialogue (QSDD). In 2013, Chang et al. proposed a QSDC protocol equipped with authentication [10]. Chou et al. proposed a bidirectional QSDC protocol for mobile network [14]. In 2014 Lai et al. proposed a quantum direct secret sharing (QDSD) scheme using fountain codes for eavesdropping check and authentication [15]; a pre-shared sequence of degrees and positions is applied to recognize the additional qubits. Hwang et al. introduced a new topic in quantum cryptography called quantum authencryption, combining quantum encryption and quantum authentication into one process for off-line communications [11]. Yang put forward a QSDC protocol without quantum memory; a stream is replaced by quantum data block to transmit quantum states [12]. Chang et al. proposed a controlled QSDC protocol; they used five-particle cluster state and quantum one time pad [6]. Zou and Qiu introduced a semiquantum secure direct communication protocol with classical Alice [13]. Gao analyzed the protocol proposed in [14] and suggested a possible improvement of it [16]. In 2015, to combat collective-dephasing noise and collective-rotation noise, Ye put forward two QD protocols [17] and a QSDD protocol [18]. Xiao and Xu proposed a high-capacity quantum secure communication scheme using either entangled pairs and an auxiliary single photon [19]. Hassanpour and Houshmand put forward a three-party controlled QSDC based on GHZ-like states which improves the efficiency of the previous ones [7]. Ma et al. presented a direct communication protocol of quantum network over noisy channel by which the bit-flip errors would be corrected using a parity matrix [20]. Chang et al. put forward a controlled deterministic secure quantum communication protocol with limited bases for preparing and measurement of qubits [21]. An experimental implementation of proposed protocol in [5], is introduced by Hu et al. [22]. Mi et al. presented a QSDC scheme using orbital angular momentum of photons to reach higher capacity and security [23]. In 2016, Uhlmann found that the anti-(conjugate) linearity plays an important role in the security of quantum cryptographic protocols [24]. Li and Yin suggested feasible quantum information processing in terms of living cells which may be applied to experimental demonstration of quantum systems [25].

As mentioned, Chang et al. put forward a QSDC and authentication protocol based on single photons, in which it is assumed that Alice and Bob have two secret strings  $ID_A$  and  $ID_B$ . Analysis of the protocol shows that it is immune against most attacks such as man-in-the-middle attack and quantum teleportation attack [10]. The current article, discovers a new attack on the recent protocol to reveal the secret information. The attack scenario is divided into two scenes. At the first scene, the attacker who is called “Oscar” tries to

obtain  $ID_B$  by sending some separate messages to Bob. At the second scene, he tries to obtain  $ID_A$  by breaking a simple XOR-encryption.

The rest of the article is as follows. Section 2 reviews Chang et al.'s protocol without any example. For a complete picture of the protocol and also related examples, we refer the readers to see [10] and references therein. Section 3 is devoted to present the new attack. In section 4, the accuracy of the new attack is studied, and the probability of perfect success after an arbitrary number of iterations is calculated in two cases: the worst case and the average case. Finally the conclusions of this article is summarized in section 5.

## 2 Review of Chang et al.'s Protocol

Throughout this section we briefly present a short review of Chang et al.'s Protocol. Alice and Bob have two secret binary strings  $(a_1, a_2, \dots, a_n) := ID_A$  and  $(b_1, b_2, \dots, b_u) := ID_B$ , already shared between themselves, which represent Alice's identity and Bob's one respectively. Suppose that Alice wants to send Bob a secret binary message  $(m_1, m_2, \dots, m_n) := M$ . Then Alice and Bob pursue the following procedure:

- Step 1 Alice encrypts  $M$  with  $ID_A$  using the simple XOR-operation and obtains  $(c_1, c_2, \dots, c_n) := C$ , where  $c_i = m_i + a_i \mod 2$ , for  $i = 1, \dots, n$ .
- Step 2 According to  $C$ , Alice creates  $n$  qubits, called  $S_C$  in the manner: if a bit of  $C$  is 0, she prepares the corresponding qubit in  $|0\rangle$  or  $|+\rangle$  state at random; otherwise she prepares the corresponding qubit in  $|1\rangle$  or  $|-\rangle$  state randomly. According to  $ID_B$ , Alice prepares  $u$  qubits, called  $S_{IDB}$  as follows: if a bit of  $ID_B$  is 0, she randomly prepares the qubit in  $|0\rangle$  or  $|1\rangle$  state; otherwise she randomly prepares the qubit in  $|+\rangle$  or  $|-\rangle$  state. Alice inserts  $S_{IDB}$  to  $S_C$  randomly which forms a new binary sequence called  $S_{C'}$  and sends it to Bob.
- Step 3 After Bob receives  $S_{C'}$ , Alice publicly announces the positions of  $S_{IDB}$  in  $S_{C'}$ . Then Bob extracts  $S_{IDB}$  and measures these photons in the correct bases according to  $ID_B$ . If a bit of  $ID_B$  is 0, he measures the corresponding qubit in  $B_Z = \{|0\rangle, |1\rangle\}$ ; otherwise,  $B_X = \{|+\rangle, |-\rangle\}$  will be applied.
- Step 4 Bob announces the state of photons in  $S_{IDB}$  which he received; the basis information is not included in this announcement. For example, Bob uses bit 0 to denote state  $|0\rangle$  and  $|+\rangle$ , and 1 for  $|1\rangle$  and  $|-\rangle$ . According to the above rule, Alice obtains the state of the initial  $S_{IDB}$ . Alice compares Bob's result with the state of initial  $S_{IDB}$ . If the error rate is low enough, Alice believes that Bob is legal and no eavesdropping exists. In this condition, the communication goes on; otherwise she interrupts it. Alice and Bob discard the bits in  $S_{IDB}$ , where the corresponding photons in  $S_{IDB}$  are not received by Bob.
- Step 5 Alice publicly announces the bases of photons in  $S_C$ . Bob measures  $S_C$  in correct bases and obtains  $C$ .
- Step 6 Bob decrypts  $C$  with  $ID_A$  bit by bit using simple XOR-operation:  $m_i = c_i + a_i \mod 2$ , for  $i = 1, \dots, n$ . In other words,  $M = C \oplus ID_A$  (Note that,

“ $\oplus$ ” is used to represent XOR-operation of two binary strings with same lengths).

Step 7 Alice takes another  $n$ -bit binary string of secret message, called  $M_1$  and starts the next transmission.

### 3 Description of The New Attack

As mentioned in previous section, the authentication inside the protocol is unidirectional, i.e. just Alice can verify Bob’s identity and demonstrates his legitimacy. Therefore Bob cannot verify the sender’s identity. Hence anyone can impersonate Alice and sends some arbitrary messages (indeed qubits) to Bob. First, we briefly explain the novel attack; the scenario of the attack is composed of two scenes:

*First scene of the attack.* Oscar prepares a binary sequence of length  $u$ , such as  $(e_1, e_2, \dots, e_u) := id_B$  (Note that  $id_B \neq ID_B$  in general). In fact  $id_B$  is a candidate for Bob’s identity binary string and changes after each session until it coincides (with high probability) on  $ID_B$ .

According to  $id_B$ , Oscar creates a sequence of qubits and obtains  $S_{idB}$  as follows: if a bit of  $id_B$  is 0, the corresponding qubit of  $S_{idB}$  is  $|0\rangle$ ; otherwise, it is  $|-\rangle$ . Next he creates a random qubit sequence as  $S_C$  and mixes it to  $S_{idB}$ , obtaining  $S_{C'}$ . Then he sends  $S_{C'}$  to Bob.

Invoking the protocol, after Bob receives  $S_{C'}$ , Oscar announces the positions of  $S_{idB}$  in  $S_{C'}$ . Then Bob measures the polarization of any photon of  $S_{idB}$  due to the corresponding bit of  $ID_B$ . The rule is that he uses  $B_Z$  basis, for corresponding “0” bits and  $B_X$  for “1” bits. Then Bob announces the state of photons in  $S_{idB}$  he received. As mentioned at step 4 of the protocol, without lose generality, assume that Bob uses bit 0 to denote state  $|0\rangle$  or  $|+\rangle$ , and 1 for  $|1\rangle$  or  $|-\rangle$ .

In other words, if a bit of the string which announced by Bob is 0, it means that the corresponding qubit he received is either  $|0\rangle$  or  $|+\rangle$ ; otherwise, it is either  $|1\rangle$  or  $|-\rangle$ .

Thus, Oscar obtains the state of the initial  $S_{idB}$ . He compares Bob’s result with the state of initial  $S_{idB}$ . If a bit of the string which announced by Bob, and the corresponding qubit of  $S_{idB}$  do not match, Oscar concludes that the corresponding bit of  $id_B$ , say  $e_i$  is wrong and changes it; otherwise the corresponding bit of  $id_B$  is probably correct, and the probability of the correctness depends on the number of session iterations. By this manner, after each iteration a new  $id_B$  replaced by the previous one. If after  $k$  iterations, no non-matching case is observed in a position, it means that the bit is correct with probability  $1 - 2^{-k}$ . Therefore, if remain  $t$  matchings after  $k$  iterations, the probability of coincident of  $id_B$  and  $ID_B$  will be  $(1 - 2^{-k})^t$ .

After Oscar obtains  $ID_B$  (with high enough probability), he can impersonate Bob.

*Second scene of the attack.* Oscar intercepts the communication between Alice and Bob. Since Oscar has  $ID_B$ , when Alice announces the positions of  $S_{IDB}$  in  $S_C$ , Oscar measures the qubits in correct bases (with high enough probability). So Alice will be deceived, and the communication goes on. Then she announces the bases of photons in  $S_C$ . Therefore Oscar has  $C$ , which is the message encrypted by  $ID_A$  using simple XOR-operation. Hence he can break it easily; see [26] and references therein.

#### 4 Numerical Exmaples and Discussion

It is clarified at step 4 of the protocol, that Alice considers the error rate when she compares Bob's result with the state of initial  $S_{IDB}$ . If it is low enough, Alice verifies the legitimacy of the receiver; see section 2. Suppose that the error rate of the quantum channel is  $\epsilon$ , and  $id_B$  differs with  $ID_B$  in  $t$  positions at the beginning of session described in the first scene of the attack. We show that after a few number of iterations, the first scene of the attack will be successful, considering  $\epsilon < 0.02$  (and even in ideal cases with high probability). After  $k$  iterations of the session,  $\lceil (1 - (3/4)^k) \cdot t \rceil$  wrong bits of  $id_B$ , will be corrected, on average (since  $(3/4)^n \rightarrow 0$  and the number of iterations is a discrete quantity, the ceiling function is used). Also it will be clear to Oscar that each of the other  $(u - t)$  bits is same as the correponding bits of  $ID_B$  with probability  $(1 - 2^{-k})$ . So, after  $k$  iterations, Oscar knows that every changed bit of latest  $id_B$  is correct, and the remaining  $x$ -bit substring is probably the same as the corresponding substring of  $ID_B$ . Hence after  $k$  iterations, if  $x$  bits do not change,  $id_B = ID_B$  with probabilty  $(1 - 2^{-k})^x$ . Table 1 shows the probability of equality  $id_B = ID_B$  after  $k$  iterations with several lengths of  $ID_B$ .

**Table 1** the probability of coincident in the worst case with  $k$  iterations.

No. of iterations $k$	The length of $ID_B$		
	(32-bit)	(64-bit)	(128-bit)
10	96.9%	93.9%	88.2%
11	98.4%	96.9%	93.9%
12	99.2%	98.4%	96.9%
13	99.6%	99.2%	98.4%

Note that table 1, shows the probability of succes in the worst-case for some number of iterations, i.e. the correction of wrong bits is not considered. But in general, the probabilty of success increases. Let  $u$  be the length of  $ID_B$ . Then there will be  $t \leq u$  wrong bits in the first candidate  $id_B$ . So, as mentioned above, after  $k$  iterations of the session,  $\lceil (1 - (3/4)^k) \cdot t \rceil$  wrong bits, on average,

will be corrected. We consider  $t = u/2$ . Table 2 shows the probability of success in the average-case.

**Table 2** the probability of coincident in the average case with  $k$  iterations.

No. of iterations $k$	The length of $ID_B$		
	(32-bit)	(64-bit)	(128-bit)
10	98.4%	96.8%	93.6%
11	99.2%	98.4%	96.8%
12	99.6%	99.1%	98.4%
13	99.8%	99.6%	99.2%

## 5 Modification of The Protocol

As explained in the previous sections, Chang et.al.'s protocol may be broken by the proposed attack. In this section we expose a proposal to reinforce the protocol. Since the loophole of the protocol is originated from “unidirectional” authentication, we propose a method for mutual authentication. This strategy can close the loophole and make the protocol more strong and resistant to the new attack. Since in the original protocol, it is assumed that Alice and Bob already sheared two secret strings  $ID_A$  and  $ID_B$ , we will also suppose that Alice and Bob have common secrets containing  $ID_A := (a_1, a_2, \dots, a_{2\ell})$ ,  $ID_B := (b_1, b_2, \dots, b_u)$  and secret key  $K_{AB} := (k_1, k_2, \dots, k_n)$ . Also the authentication procedure and sending the secret message will be based on single photons and quantum memory as in the original protocol. The modified protocol to send the message  $M := (m_1, m_2, \dots, m_n)$  is as follows:

- Step 1 Alice encrypts the message  $M$  by  $K_{AB}$  using XOR-operation, so that she obtains  $C = M \oplus K_{AB}$ .
- Step 2 Exactly like the original protocol, Alice prepares  $S_C$  and  $S_{IDB}$ . In addition, She creates  $\ell$  qubits, called  $S_{IDA}$ , according to  $ID_A$ . The procedure of creating  $S_{IDA}$  is that Alice randomly chooses  $\ell$  bits of  $ID_A$  as an ordered basis string by the rule:  $0 \equiv B_Z$  and  $1 \equiv B_X$ . The remaining  $\ell$  bits are used as an ordered qubit string measured (after reordering if necessary) in the corresponding bases by the rule:  $0 \equiv |0\rangle$  or  $|+\rangle$  and  $1 \equiv |1\rangle$  or  $|-\rangle$  due to the corresponding basis. Then Alice inserts  $S_{IDA}$  and  $S_{IDB}$  randomly to  $S_C$ , obtaining  $S'_C$ , and sends it to Bob.
- Step 3 After Bob receives  $S'_C$ , Alice announces the positions of  $S_{IDA}$ ,  $t$  bits of  $ID_A$  used for sent qubits by order, and  $S_{IDB}$ . Then Bob extracts the qubits of  $S_{IDA}$  and  $S_{IDB}$ .
- Step 4 Bob measures the qubits of  $S_{IDA}$  and checks the legitimacy of Alice. If she is legal, then Bob measures the qubits of  $S_{IDB}$  and sends the result to Alice exactly like the original protocol; otherwise he interrupts it.

- Step 5 Alice checks the legitimacy of Bob as described in [10]. If he is legal, she announces the bases of  $S_C$  qubits; otherwise she interrupts the communication.
- Step 6 Bob decrypts message  $C$  by  $K_{AB}$ , using XOR-operation to obtain the plaintext message  $M$ .

## 6 Security Analysis of The Modified Protocol

As analyzed in [10], the original protocol is secure against most attacks such as man in the middle attack and quantum teleportation attack, but the proposed attack can break the protocol and the attacker can obtain  $ID_A$  and  $ID_B$  which are secrets already shared between Alice and Bob. The modification of the protocol which is proposed in the previous section, can fill the loophole of the protocol. Since the loophole is emanated from unidirectional authentication, applying the bidirectional authentication proposed in the previous section fills it up. Formal speaking, suppose that attacker Oscar tries to impersonate Alice, and sends Bob a message. When Bob receives the message, Oscar should announce the positions of  $\ell$  bits of  $ID_A$ , which show his legitimacy. So he cannot impersonate Alice, unless he accesses to  $ID_A$ . Therefore any non-legal person who does not access to  $ID_A$ , cannot deceive Bob. On the other hand, if an adversary tries to impersonate Bob, after he receives the message, obtains no information about  $ID_A$ . More precisely, it is possible that an adversary receives the message, since Bob proves his legitimacy after Alice. But there is nothing to worry about, because when Alice announces the positions of the  $\ell$  verification bits, she does not declare the correct bases of them. So the adversary obtains no information about  $ID_A$ . Moreover he cannot impersonate Bob (unless he accesses to  $ID_B$ ). Therefore when Alice checks the legitimacy of the receiver, would not be deceived and interrupts the communication. In this situation, reordering of  $\ell$  verification bits is necessary to retry sending the message. To reach more security it is suggested that a derangement on  $\ell$  verification bits be applied. Note that repetition of sending message to a non-legal person, allows him to guess  $ID_A$  applying a same method described in the first scene of the attack; see section 3. Hence, one can deduce if a qubit of  $ID_A$  loses once because of noisy channel or eavesdropping, it can be sent in the next transmissions safely, provided that its corresponding basis index, changes. However if a fixed qubit loses twice, it must be discarded.

Clearly the modified protocol defeat man-in-the-middle attack and quantum teleportation attack as the original protocol can.

## 7 Conclusion

We have demonstrated that Chang et al.'s protocol, is vulnerable to a specific attack, which is described in this article. Our attack scenario is divided into two scenes. Since Bob cannot check Alice's legitimacy, at the first scene, Oscar impersonates Alice. Then he chooses a candidate binary string for  $ID_B$ ,

and corrects the wrong bits of it by sending a message several times to Bob, due to the original protocol. After a few iterations, he obtains  $ID_B$ , with high enough probability. At the second scene, Oscar intercepts the transmission between Alice and Bob, and impersonates Bob easily. Then he can receive any message which is sent by Alice, measure the qubits in the correct bases, and finally obtain  $ID_A$  by breaking a simple XOR-encryption. Since the loophole of the protocol is originated from the unidirectional authentication, the mutual authentication is suggested to defeat the attack. Furthermore the modified protocol is secure against all attacks which the original one is.

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International Journal of Quantum Information  
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## An Attack on “Quantum Secure Direct Communication and Authentication Protocol With Single Photons”

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Received Day Month Year

Revised Day Month Year

In a recent article, Chang et al. proposed a quantum secure direct communication protocol using single photons (Chinese Sci Bull, 58: 4571-4576). The protocol is equipped with authentication. In this article we present a novel attack on the protocol that can determine the secret key which is already shared between Alice and Bob.

*Keywords:* quantum secure direct communication; attack; authentication; single photons.

### 1. Introduction

Over the last decades, quantum cryptography plays a significant role in abstract theory of information and communication security. It is divided into some major research topics, such as QKD <sup>a</sup>, QSS <sup>b</sup>, QIA <sup>c</sup>, etc. which have developed from their first publications Refs. 1–3 respectively, until now.

A new topic of quantum cryptography which has been studied at depth, comprehensively recently, is quantum secure direct communication, known as QSDC. The goal of QSDC is to convey a secret message directly without a key generating session to encrypt the message. Like many other topics of quantum cryptography, there are two approaches to research on QSDC: quantum entanglement Refs. 4, 5 and single photons Refs. 6, 7. In 2013, Chang et al. proposed a QSDC protocol<sup>8</sup> equipped with authentication, based on single photons. In the article, it is assumed that Alice and Bob have two secret strings  $ID_A$  and  $ID_B$ . As mentioned in the recent article, the protocol is immune against most attacks such as man-in-the-middle attack and quantum teleportation attack.

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<sup>a</sup>quantum key distribution

<sup>b</sup>quantum secret sharing

<sup>c</sup>quantum identity authentication

The current article, presents a new attack on the recent protocol to reveal the secret information. The attack scenario is divided into two scenes. At the first scene the attacker who is called “Oscar” tries to obtain  $ID_B$  by sending some separate messages to Bob. At the second scene he tries to obtain  $ID_A$  by breaking a simple XOR-encryption.

The rest of the article is as follows. Section 2 reviews Chang et al.’s protocol without any example. For a complete picture of the protocol and also related examples, we refer the readers to see Ref. 8 and references therein. Section 3 is devoted to present the new attack. In section 4, the accuracy of the new attack is studied, and the probability of perfect success after an arbitrary number of iterations is calculated in two cases: the worst case and the average case. Finally the conclusions of this article is summarized in section 5.

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Step 1. Alice encrypts  $M$  with  $ID_A$  using the simple XOR-operation and obtains  $(c_1, c_2, \dots, c_n) := C$ , where  $c_i = m_i + a_i \pmod{2}$ , for  $i = 1, \dots, n$ .

Step 2. According to  $C$ , Alice creates  $n$  qubits, called  $S_C$  in the manner: if the bit of  $C$  is 0, she prepares the corresponding qubit in  $|0\rangle$  or  $|+\rangle$  state at random; otherwise she prepares the corresponding qubit in  $|1\rangle$  or  $|-\rangle$  state randomly. According to  $ID_B$ , Alice prepares  $u$  qubits, called  $S_{IDB}$  as follows: if a bit of  $ID_B$  is 0, she randomly prepares the qubit in  $|0\rangle$  or  $|1\rangle$  state; otherwise she randomly prepares the qubit in  $|+\rangle$  or  $|-\rangle$  state. Alice inserts  $S_{IDB}$  to  $S_C$  randomly (forms sequence  $S_{C'}$ ) and sends it to Bob.

Step 3. After Bob receives  $S_{C'}$ , Alice publicly announces the positions of  $S_{IDB}$  in  $S_{C'}$ . Then Bob extracts  $S_{IDB}$  and measures these photons in the correct bases according to  $ID_B$ . If a bit of  $ID_B$  is 0, he measures the corresponding qubit in  $B_Z = \{|0\rangle, |1\rangle\}$ ; otherwise,  $B_X = \{|+\rangle, |-\rangle\}$  will be applied.

Step 4. Bob announces the state of photons in  $S_{IDB}$  which he received; the basis information is not included in this announcement. For example, Bob uses bit 0 to denote state  $|0\rangle$  and  $|+\rangle$ , and 1 for  $|1\rangle$  and  $|-\rangle$ . According to the above rule, Alice obtains the state of the initial  $S_{IDB}$ . Alice compares Bob’s result with the state of initial  $S_{IDB}$ . If the error rate is low enough, Alice believes that Bob is legal and no eavesdropping exists. In this condition, the communication goes on; otherwise she interrupts it. Alice and Bob discard the bits in  $S_{IDB}$ , where the corresponding photons in  $S_{IDB}$  are not received by Bob.

Step 5. Alice publicly announces the bases of photons in  $S_C$ . Bob measures  $S_C$

in correct bases and obtains  $C$ .

Step 6. Bob decrypts  $C$  with  $ID_A$  bit by bit using simple XOR-operation:  $m_i = c_i + a_i \bmod 2$ , for  $i = 1, \dots, n$ . In other words,  $M = C \oplus ID_B$  (Note that, “ $\oplus$ ” is used to represent XOR-operation).

Step 7. Alice takes another  $n$ -bit binary string of secret message, called  $M_1$  and starsts the next transmission.

### 3. Description of The New Attack

As mentioned in previous section, the authentication inside the protocol is direct, i.e. just Alice can verify Bob’s identity and demonstrates his legitimacy. Therefore Bob cannot verify the sender’s identity. Hence anyone can impersonate Alice and sends some arbitrary messages (indeed qubits) to Bob. First, we briefly explain the novel attack; the scenario of the attack is composed of two scenes:

#### 3.1. First scene of the attack

Oscar prepares a binary sequence of length  $u$ , called  $id_B(e_1, e_2, \dots, e_u)$ . (Note that  $id_B \neq ID_B$  in general). In fact  $id_B$  is a candidate for Bob’s identity binary string and changes after each session untill coincides (with high probability) on  $ID_B$ .

According to  $id_B$ , Oscar creates a sequeence of qubits and obtains  $S_{idB}$  as follows: if a bit of  $id_B$  is 0, the corresponding qubit of  $S_{idB}$  is  $|0\rangle$ ; otherwise, it is  $|-\rangle$ . Next he creates a random qubit sequence as  $S_C$  and mixes it to  $S_{idB}$ , obtaining  $S_{C'}$ . Then he sends  $S_{C'}$  to Bob.

Invoking the protocol, after Bob receives  $S_{C'}$ , Oscar announces the positions of  $S_{idB}$  in  $S_{C'}$ . Then Bob measures the polarization of any photon of  $S_{idB}$  due to the corresponding bit of  $ID_B$ . The rule is that he uses  $B_Z$  basis, for corresponding “0” bits and  $B_X$  for “1” bits.

Then Bob announces the state of photons in  $S_{idB}$  he received. As mentioned at step 4 of the protocol, without lose generality, assume that Bob uses bit 0 to denote state  $|0\rangle$  or  $|+\rangle$ , and 1 for  $|1\rangle$  or  $|-\rangle$ .

In other words, if a bit of the string which announced by Bob is 0, it means that the corresponding qubit he received is either  $|0\rangle$  or  $|+\rangle$ ; otherwise, it is either  $|1\rangle$  or  $|-\rangle$ .

Thus, Oscar obtains the state of the initial  $S_{idB}$ . He compares Bob’s result with the state of initial  $S_{idB}$ . If a bit of the string which announced by Bob, and the corresponding qubit of  $S_{idB}$  do not match, Oscar concludes that the corresponding bit of  $id_B$ , say  $e_i$  is wrong and changes it; otherwise the corresponding bit of  $id_B$  is probably correct, and the probability of the correctness depends on the number of session iterations. By this manner, after each iteration a new  $id_B$  replaced by the previous one. If after  $k$  iterations, no non-matching case is observed in a position, it means that the bit is correct with probability  $1 - 2^{-k}$ . Therefore, if remain  $t$  matchings after  $k$  iterations, the probability of coincident of  $id_B$  and  $ID_B$  will be  $(1 - 2^{-k})^t$ .

After Oscar obtains  $ID_B$  (with high enough probability), he can impersonate Bob.

### 3.2. Second scene of the attack

Oscar intercepts the communication between Alice and Bob. Since Oscar has  $ID_B$ , when Alice announces the positions of  $S_{ID_B}$  in  $S_{C'}$ , Oscar measures the qubits in correct bases (with high enough probability). So Alice will be deceived, and the communication goes on. Then she announces the bases of photons in  $S_C$ . Therefore Oscar has  $C$ , which is the message encrypted by  $ID_A$  using simple XOR-operation. Hence he can break it easily; see Ref. 9 and references therein.

## 4. Numerical Exmaples and Discussion

It is clarified at step 4 of the protocol, that Alice considers the error rate when she compares Bob's result with the state of initial  $S_{ID_B}$ . If it is low enough, Alice verifies the legitimacy of the receiver; see section 2. Suppose that the phase error rate of the channel is  $\epsilon$ , and  $id_B$  differs with  $ID_B$  in  $t$  positions at the beginning of session described in the first scene of the attack. We show that after a few number of iterations, the first scene of the attack will be successful, for  $\epsilon < 0.05$ . After  $k$  iterations of the session,  $\lceil (1 - (3/4)^k).t \rceil$  wrong bits of  $id_B$ , will be corrected, on average (since  $(3/4)^n \rightarrow 0$  and the number of iterations is a discrete quantity, the ceiling function is used). Also it will be clear to Oscar that each of the other  $(u - t)$  bits is same as the correponding bit of  $ID_B$  with probability  $(1 - 2^{-k})$ . So, after  $k$  iterations, Oscar knows that every changed bit of latest  $id_B$  is exactly correct, and the remaining  $x$ -bit substring is probably the same as the corresponding substring of  $ID_B$ . Hence after  $k$  iterations, if  $x$  bits do not change,  $id_B = ID_B$  with probability  $(1 - 2^{-k})^x$ . Table 1 shows the probability of equality  $id_B = ID_B$  after  $k$  iterations with several lengths of  $ID_B$ .

Table 1. the probability of coincident in the worst case with  $k$  iterations.

The number iterations $k$	The length of $ID_B$		
	(32-bit)	(64-bit)	(128-bit)
10	96.9%	93.9%	88.2%
11	98.4%	96.9%	93.9%
12	99.2%	98.4%	96.9%
13	99.6%	99.2%	98.4%

Note that table 1, shows the probability of succes in the worst-case for some number of iterations i.e. the correction of wrong bits is not considered. But in general, the probability of success increases. Let  $u$  be the length of  $ID_B$ . Then there will be  $t \leq u$  wrong bits in the first candidate  $id_B$ . So, as mentioned above, after  $k$

iterations of the session,  $\lceil(1 - (3/4)^k).t\rceil$  wrong bits, on average, will be corrected. We consider  $t = u/2$ . Table 2 shows the probability of success in the average-case.

Table 2. the probability of coincident in the average case with  $k$  iterations.

The number iterations $k$	The length of $ID_B$		
	(32-bit)	(64-bit)	(128-bit)
10	98.4%	96.8%	93.6%
11	99.2%	98.4%	96.8%
12	99.6%	99.1%	98.4%
13	99.8%	99.6%	99.2%

The bit error rate of the transmission channel has not been considered in the above examples. If it is denoted by  $\delta$ , since usually  $\delta < \epsilon$ , one can check the results, which are shown in table 1 and table 2, would not changed noticeably.

## 5. Conclusion

We have demonstrated that Chang et.al’s protocol, is vulnerable to a specific attack, which is described in this article. Our attack scenario is divided into two scenes. Since Bob cannot check Alice’s legitimacy, in the first scene, Oscar impersonates Alice. Then he chooses a candidate binary string for  $ID_B$ , and corrects the wrong bits of it by sending a message several times to Bob, due to the original protocol. After a number of iterations, he obtains  $ID_B$ , with high enough probability. In the second scene, Oscar intercepts the transmission between Alice and Bob, and impersonates Bob easily. Then he can receive any message which is sent by Alice, measure the qubits in the correct bases, and finally obtain  $ID_A$  by breaking a simple XOR-encryption.

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6 *A. Amerimehr & M. H. Dehkordi*

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